

Hurwitz Polynomial:

$$H(s) = \frac{N(s)}{D(s)}$$

* Conditions of Hurwitz Polynomial:

- 1) All Coefficients are +ve
- 2) All Coefficients are exist.

ex 1:

Test whether the following polynomial is Hurwitz or not?

$$P(s) = s^4 + s^3 + 5s^2 + 3s + 4$$

$$\Rightarrow \text{even part } m(s) = s^4 + 5s^2 + 4$$

$$\Rightarrow \text{odd part } n(s) = s^3 + 3s$$

$$\therefore \text{degree of } m(s) > \text{degree of } n(s)$$

$$\frac{n(s)}{m(s)}$$

All quotient are +ve

$\therefore$ it is Hurwitz

$$\left\{ \begin{array}{l} q_1 = s \\ +ve \\ q_2 = \frac{s}{2} \\ +ve \\ q_3 = 2s \\ +ve \\ q_4 = \frac{s}{4} \\ +ve \end{array} \right.$$

$$\begin{array}{r} \boxed{s^3 + 3s} \overline{) s^4 + 5s^2 + 4} \\ \underline{s^4 + 3s^2} \\ 2s^2 + 4 \\ \boxed{2s^2 + 4} \overline{) 2s^2 + 4} \\ \underline{2s^2 + 4} \\ 0 \end{array}$$

test Hurwitz or not?

(2)

$$P(s) = s^4 + s^3 + 2s^2 + 3s + 2$$

> even part $m(s) = s^4 + 2s^2 + 2$

> odd part $n(s) = s^3 + 3s$

$$q_1 = s$$

+ve

$$q_2 = -s$$

-ve

$$\begin{array}{r} s^3 + 3s \overline{) s^4 + 2s^2 + 2} \\ \underline{s^4 + 3s^2} \\ -s^2 + 2 \overline{) s^3 + 3s} \end{array}$$

∴ it is not Hurwitz.

ex 3:

$$P(s) = s^7 + 2s^6 + 2s^5 + s^4 + 4s^3 + 8s^2 + 8s + 4$$

⇒ even part $m(s) = 2s^6 + s^4 + 8s^2 + 4$

⇒ odd part $n(s) = s^7 + 2s^5 + 4s^3 + 8s$

$$q_1 = \frac{1}{2}s$$

+ve

$$q_2 = \frac{4}{3}s$$

+ve

$$q_3 = \frac{3}{2}s$$

+ve

$$\begin{array}{r} 2s^6 + s^4 + 8s^2 + 4 \overline{) s^7 + 2s^5 + 4s^3 + 8s} \\ \underline{s^7 + \frac{1}{2}s^5 + 4s^3 + 2s} \\ \frac{3}{2}s^5 + 6s \overline{) 2s^6 + s^4 + 8s^2 + 4} \\ \underline{\frac{3}{2}s^5 + 6s^3} \\ 2s^6 + s^4 + 8s^2 + 4 \\ \underline{2s^6 + 6s^3} \\ s^4 + 4 \overline{) \frac{3}{2}s^5 + 6s} \\ \underline{\frac{3}{2}s^5 + 6s^3} \\ 0 + 0 \end{array}$$

There is only 3 quotient only ∴ There is Common factor

$$\begin{array}{r}
 S^3 + 2S^2 + 2S + 1 \quad \dots \\
 \hline
 S^4 + 4 \overline{) S^7 + 2S^6 + 2S^5 + S^4 + 4S^3 + 8S^2 + 8S + 4} \\
 \underline{S^7} + 4S^3 \\
 2S^6 + 2S^5 + S^4 + 8S^2 + 8S + 4 \\
 \underline{2S^6} + 8S^2 \\
 2S^5 + S^4 + 8S + 4 \\
 \underline{2S^5} + 8S \\
 S^4 + 4 \\
 \underline{S^4 + 4} \\
 0 + 0
 \end{array}$$

$$\therefore f(s) = (S^4 + 4)(S^3 + 2S^2 + 2S + 1)$$

$$\text{but } S^4 + 4 = 0$$

$$S^2 = \pm 2j$$

$\therefore$ there is multipoles at j -axis

$\therefore$ not Hurwitz

Test Hurwitz or not?

[4]

$$* F(s) = s^3 + 2s^2 + 3s + 6$$

$$m(s) = 2s^2 + 6$$

$$n(s) = s^3 + 3s$$

$$q_1 \rightarrow \boxed{\frac{s}{2}} \quad \begin{array}{r} 2s^2 + 6 \overline{) s^3 + 3s} \\ \underline{s^3 + 3s} \\ 0 \end{array}$$

+ve

* There must be 3 quotients, since there is only 1q $\therefore$ there must be a common factor

$$\begin{array}{r} \frac{1}{2}s + 1 \\ 2s^2 + 6 \overline{) s^3 + 2s^2 + 3s + 6} \\ \underline{s^3 + 3s} \\ 2s^2 + 6 \\ \underline{2s^2 + 6} \\ 0 \end{array}$$

$$F(s) = (2s^2 + 6)(\frac{1}{2}s + 1)$$

$$\begin{array}{l|l} 2s^2 + 6 = 0 & \frac{1}{2}s + 1 = 0 \\ \hline \therefore s = \pm j\sqrt{3} & s = -2 \end{array}$$

$\therefore$ poles are in LHP & has a single pole at j-axis
 $\therefore$ it is Hurwitz

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$$P(s) = N(s) + D(s)$$

$$m(s) = 3s^2 + 41$$

$q_1 \rightarrow \boxed{\frac{5}{3}}$
 +ve

$q_2 \rightarrow \boxed{\frac{-95}{29}}$
 -ve

$3S^2 + 41 \mid \begin{array}{r} S^3 + 4S \\ S^3 + \frac{41}{3}S \\ \hline -\frac{29}{3}S \end{array} \quad 3S^2 + 41$

$\therefore$ it is non-Hurwitz

Positive real function (PRF)

[1]

⇒ To be PRF the following requirements are fulfilled:

1) $m_1 m_2 - n_1 n_2 \geq 0 \quad \forall \omega$ → الشرط لازم يصدق لكل قيم ω

m_1 : even part of $N(s)$

n_1 : odd part of $N(s)$

m_2 : even part of $D(s)$

n_2 : odd part of $D(s)$

All Coeff. are real & +ve

all poles lie in LHP

must be Hurwitz

$$F(s) = \frac{N(s)}{D(s)}$$

$$P(s) = N(s) + D(s)$$

Properties of PRF:

1) if $F(s)$ is PRF $\therefore \frac{1}{F(s)}$ is also PRF "if $Z(s)$ is PRF $\therefore Y(s)$ is PRF"

The sum of PRFs is PRF but the difference is not necessary.

if $Z_1(s)$ & $Z_2(s)$ are PRF

$$F(s) = Z_1(s) + Z_2(s) \therefore F(s) \text{ is PRF}$$

$$P(s) = Z_1(s) - Z_2(s) \therefore P(s) \text{ is not necessary PRF}$$

Test PRF or not & give reasons.

[2]

1:

$$F(s) = \frac{s^2 + 5s}{s^4 + 2s^2 + 1} = \frac{s^2 + 5s}{(s^2 + 1)^2} \rightarrow \text{not PRF bec. it has multipoles at } j\omega \text{ axis}$$

2:

$$Y(s) = \frac{9s + 2}{s + 2}$$

$$s + 2 = 0$$

$$\therefore s = -2$$

has no poles at RHP

$$m_1(s) = 2, \quad n_1(s) = 9s$$

$$m_2(s) = 2, \quad n_2(s) = s$$

$$m_1 m_2 - n_1 n_2 \Big|_{s=j\omega} \geq 0$$

$$4 - 9s^2 \Big|_{s=j\omega} \geq 0$$

$$4 + 9\omega^2 \geq 0 \text{ for all } \omega (+\omega)$$

$\therefore F(s)$ is PRF.

x3:

$$F(s) = \frac{2s^3 + 3s^2 + 3s + 3}{s^2 + 1}$$

[3]

$$P(s) = 2s^3 + 4s^2 + 3s + 4$$

$$m(s) = 4s^2 + 4$$

$$n(s) = 2s^3 + 3s$$

$$\begin{array}{l} \text{we } q_1 = \frac{1}{2} s \\ \text{we } q_2 = 4s \\ \text{we } q_3 = \frac{1}{4} s \end{array} \quad \begin{array}{r} 4s^2 + 4 \overline{) 2s^3 + 3s} \\ \underline{2s^3 + 2s} \\ 4s^2 + 4 \\ \underline{4s^2} \\ 4s \\ \underline{4s} \\ 0 \end{array}$$

∴ Herwitz

$$n_1 = 3s^2 + 3, \quad n_1 = 2s^3 + 3s$$

$$n_2 = s^2 + 1, \quad n_2 = 0$$

$$m_1 m_2 - n_1 n_2 \geq 0 \quad \forall \omega$$

$$3s^4 + 6s^2 + 3 \geq 0 \quad \forall \omega$$

$$3\omega^4 - 6\omega^2 + 3 \geq 0 \quad \forall \omega$$

∴ F(s) is PRF

Q4: Determine the Condition for biquadratic function to be PRF (4)

$$f(s) = \frac{s^2 + a_1s + a_0}{s^2 + b_1s + b_0}$$

a_0, a_1, b_0, b_1 are +ve & real

$$m_1 = s^2 + a_0, \quad n_1 = a_1s$$

$$m_2 = s^2 + b_0, \quad n_2 = b_1s$$

$$m_1m_2 - n_1n_2 \geq 0$$

$$(s^2 + a_0)(s^2 + b_0) - a_1b_1s^2 \geq 0$$

$$s^4 + (a_0 + b_0)s^2 + a_0b_0 - a_1b_1s^2 \geq 0$$

$$s^4 + (a_0 + b_0 - a_1b_1)s^2 + a_0b_0 \geq 0$$

$$w^4 - (a_0 + b_0 - a_1b_1)w^2 + a_0b_0 \geq 0$$

assume $w^2 = x^2$
 $\therefore w^2 = x$

$$x^2 - (a_0 + b_0 - a_1b_1)x + a_0b_0 \geq 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{a_0 + b_0 - a_1b_1 \pm \sqrt{[-(a_0 + b_0 - a_1b_1)]^2 - 4a_0b_0}}{2}$$

$$(a_0 + b_0 - a_1b_1)^2 > 4a_0b_0$$

$$a_0 + b_0 - a_1b_1 > 2\sqrt{a_0b_0}$$

$$a_0 + b_0 - 2\sqrt{a_0b_0} > a_1b_1$$

$$(\sqrt{a_0} - \sqrt{b_0})^2 > a_1b_1$$

$$b^2 \geq 4ac$$

Condition to make sure that w is real

Q5:

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$$F(s) = \frac{s^2 + 2s + a}{s^2 + 3s + 2}$$

Find a to be PRF

$$m_1 = s^2 + a, \quad n_1 = 2s$$

$$m_2 = s^2 + 2, \quad n_2 = 3s$$

$$m_1 m_2 - n_1 n_2 \geq 0$$

$$(s^2 + a)(s^2 + 2) - 6s^2 \geq 0$$

$$s^4 + (a-4)s^2 + 2a \geq 0$$

$s = j\omega$

$$\omega^4 - (a-4)\omega^2 + 2a \geq 0$$

→ Real, +ve

$$b^2 \geq 4ac$$

$$[-(a-4)]^2 \geq 4 \times 2a$$

$$\therefore (a-4)^2 \geq 8a$$

$$a-4 \geq 2\sqrt{2a}$$

Ex 6:

$$F(s) = \frac{s+a}{s^2+bs+c}$$

[6]

a, b, c are +ve & real

$$m_1 = a, n_1 = s$$

$$m_2 = s^2 + c, n_2 = bs$$

$$m_1 m_2 - n_1 n_2 \Big|_{s=j\omega} \geq 0$$

$$as^2 + ac - bs^2 \Big|_{s=j\omega} \geq 0$$

$$(a-b)s^2 + ac \Big|_{s=j\omega} \geq 0$$

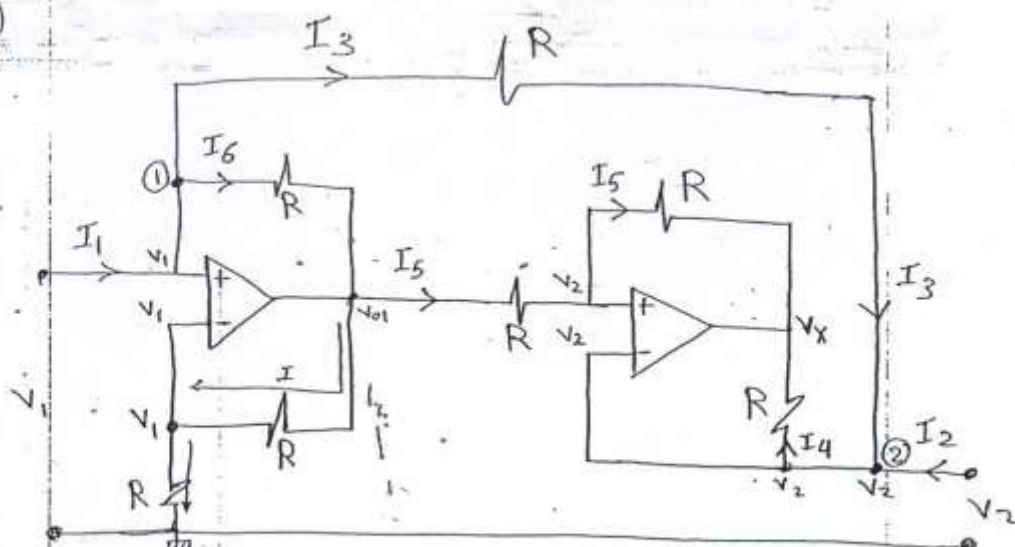
$$-(a-b)\omega^2 + ac \geq 0$$

$$(b-a)\omega^2 + ac \geq 0$$

$$b-a > 0 \therefore b > a$$

$$c > 0$$

(1)



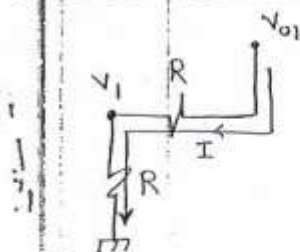
apply KCL at node (2) :-

$$I_4 = I_2 + I_3$$

$$I_2 = I_4 - I_3 = \frac{V_2 - V_X}{R} - \frac{V_1 - V_2}{R}$$

$$I_2 = \frac{V_2 - V_X}{R} - \frac{V_1 - V_2}{R} \quad (1)$$

$$I_5 = \frac{V_{01} - V_2}{R} = \frac{V_2 - V_X}{R} \quad (2)$$



$$\frac{V_{o1} - V_1}{R} = \frac{V_1}{R}$$

$$2V_1 = V_{01} \quad (3)$$

Substitute from (3) in (2) :-

$$2V_1 - V_2 = V_2 - V_X$$

$$V_x = 2V_2 - 2V_1 \quad (4)$$

Substitute from (4) in (1) :-

$$I_2 = \frac{V_2}{R} - \frac{2V_2}{R} + \frac{2V_1}{R} - \frac{V_1}{R} + \frac{V_2}{R}$$

$$\therefore I_2 = \frac{V_1}{R}$$

$$V_1 = I_2 R$$

apply KCL at node ① :-

$$I_1 = I_6 + I_3$$

$$I_1 = \frac{V_1 - V_{01}}{R} + \frac{V_1 - V_{12}}{R}$$

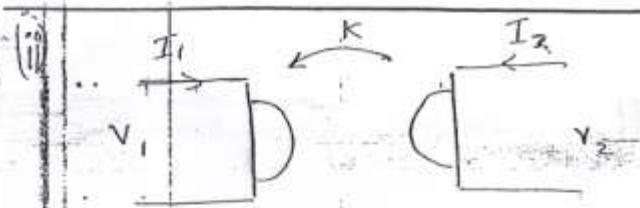
$$\% \text{B} \quad 2\gamma_1 = \gamma_{01}$$

$$\therefore I_1 = \frac{V_1}{R} - \frac{2V_1}{R} + \frac{V_1}{R} - \frac{V_2}{R_2}$$

$$I_1 = -\frac{V_2}{R}$$

$$\therefore T = \begin{bmatrix} 0 & -R \\ -1/R & 0 \end{bmatrix}$$

the circuit realize a given



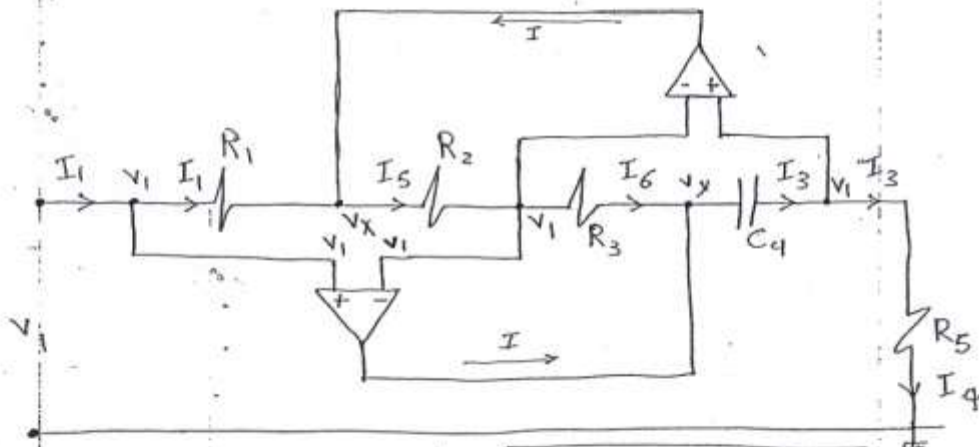
$$V_1 = I_2 k$$

$$V_2 = -I_1 k$$

$$\therefore I_1 = -\frac{1}{k} V_2$$

$$\therefore T = \begin{bmatrix} 0 & -k \\ -1/k & 0 \end{bmatrix}$$

2



$$I_1 = \frac{V_1 - V_x}{R_1} \quad (1)$$

$$I_5 = I_6$$

$$\therefore \frac{V_x - V_1}{R_2} = \frac{V_1 - V_y}{R_3} \quad (2)$$

$$I_3 = \frac{V_y - V_1}{\frac{1}{sC_4}} = \frac{V_1}{R_5}$$

$$\therefore (V_y - V_1) sC_4 = \frac{V_1}{R_5}$$

$$V_y - V_1 = \frac{V_1}{sC_4 R_5}$$

$$\therefore V_1 - V_y = -\frac{V_1}{sC_4 R_5} \quad (3)$$

from (3) in (2) :-

$$\frac{V_x - V_1}{R_2} = -\frac{V_1}{sC_4 R_5 R_3}$$

$$\therefore V_x = V_1 - \frac{V_1 R_2}{sC_4 R_5 R_3} \quad (4)$$

from (4) in (1) :-

$$\therefore I_1 = \cancel{\frac{V_1}{R_1}} - \cancel{\frac{V_1}{R_1}} + \frac{V_1 R_2}{S C_4 R_5 R_3 R_1}$$

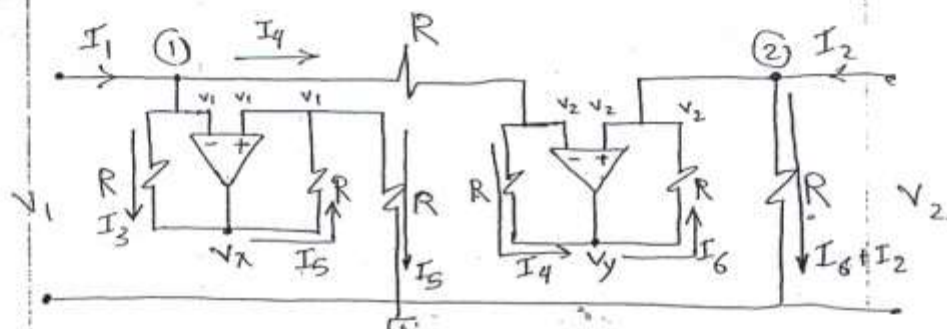
$$\therefore Z_{in} = \frac{V_1}{I_1} = \frac{S C_4 R_5 R_3 R_1}{R_2} = S L$$

$$\text{where :- } L = \frac{C_4 R_5 R_3 R_1}{R_2}$$

this circuit is equivalent to a grounded inductor.



4)



apply kcl at node (1) :-

$$I_1 = I_4 + I_3$$

$$I_1 = \frac{V_1 - V_x}{R} + \frac{V_1 - V_2}{R} \quad (1)$$

$$I_5 = \frac{V_x - V_1}{R} = \frac{V_1}{R}$$

$$\therefore 2V_1 = V_x \quad (2)$$

from (2) in (1) :-

$$I_1 = \cancel{\frac{V_1}{R}} - \cancel{\frac{2V_1}{R}} + \cancel{\frac{V_1}{R}} - \frac{V_2}{R}$$

$$\therefore I_1 = -\frac{V_2}{R}$$

apply kcl at node (2) :-

$$I_6 + I_2 = \frac{V_2}{R}$$

$$\therefore I_2 + \frac{V_y - V_2}{R} = \frac{V_2}{R}$$

$$\therefore I_2 + \frac{V_y}{R} = \frac{2V_2}{R} \quad (3)$$

$$I_4 = \frac{V_1 - V_2}{R} = \frac{V_2 - V_y}{R}$$

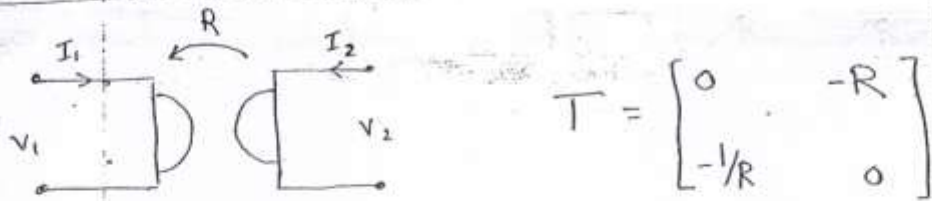
$$\therefore V_y = 2V_2 - V_1 \quad (4)$$

from (4) in (3) :-

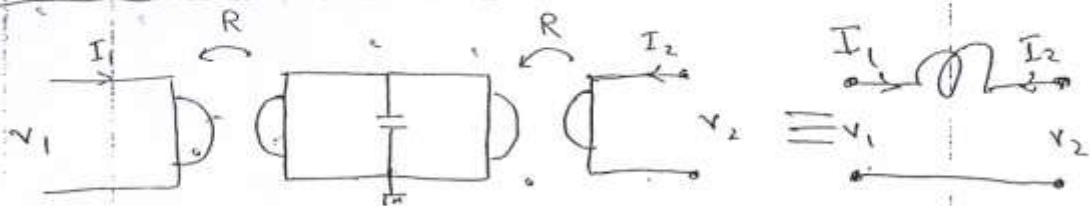
$$I_2 + \frac{2V_2 - V_1}{R} = \frac{2V_2}{R}$$

$$I_2 = \frac{V_1}{R} \Rightarrow I_2 R = V_1$$

This circuit realize a gyrator :-

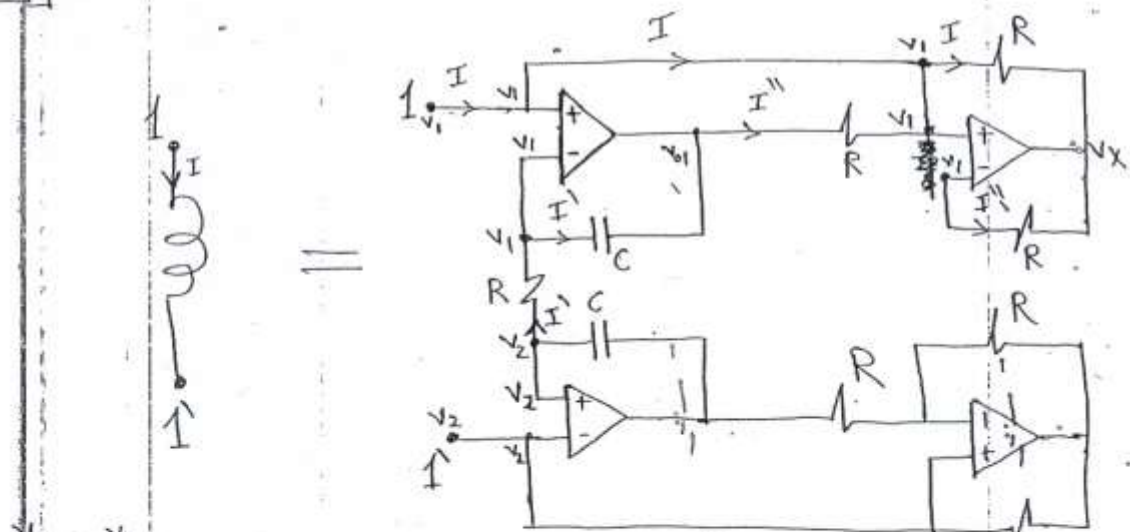


we can simulate a floating inductor :-



the total number of element are needed to simulate the floating inductor = 4 opamp + 14 R + 1 C = 19

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$$\frac{V_1 - V_x}{R} = I$$

$$\frac{V_1 - V_x}{R} = I''$$

$$\therefore I = I''$$

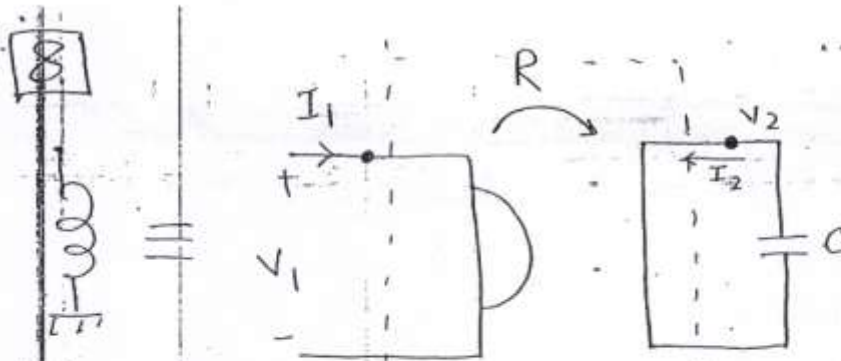
$$I' = \frac{V_2 - V_1}{R} = (V_1 - V_{o1}) SC$$

$$I'' = \frac{V_{o1} - V_1}{R} = I$$

$$\therefore \frac{V_1 - V_{o1}}{R} = \frac{V_2 - V_1}{R^2 SC} = -I$$

$$\therefore \frac{V_1 - V_2}{R^2 SC} = I$$

$$\therefore \frac{V_1 - V_2}{I} = R^2 SC = SL$$



from gyrator :-

$$V_1 = -I_2 R \rightarrow (1)$$

$$V_2 = R I_1 \rightarrow (2)$$

$$\therefore I_1 = \frac{1}{R} V_2 \rightarrow (2)$$

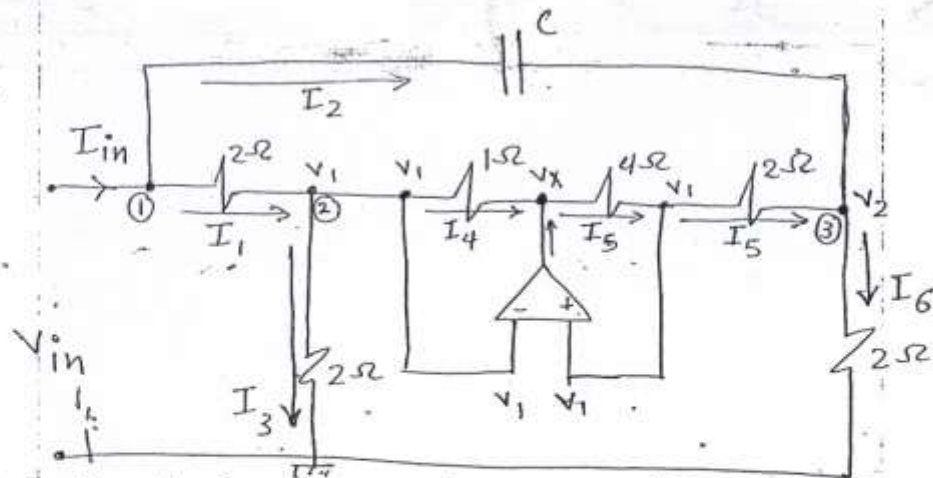
$$V_2 = -I_2 \frac{1}{sC} \rightarrow (3)$$

$$\therefore Z_{in} = \frac{V_1}{I_1} = \frac{-I_2 R}{V_2/R} = -\frac{I_2}{V_2} R^2$$

$$\therefore Z_{in} = \frac{I_2}{I_2 \frac{1}{sC}} R^2 = R^2 sC = sL$$

$$L = R^2 C$$

[B]



apply KCL at node ① :-

$$I_{in} = I_1 + I_2 = \frac{v_{in} - v_1}{2} + \frac{v_{in} - v_2}{1/s_c}$$

$$= \frac{1}{2} v_{in} - \frac{1}{2} v_1 + v_{in} s_c - v_2 s_c$$

$$\therefore I_{in} = v_{in} \left[\frac{1}{2} + s_c \right] - \frac{1}{2} v_1 - v_2 s_c \quad (1)$$

apply KCL at node ② :-

$$I_1 = I_3 + I_4 = \frac{v_1}{2} + \frac{v_1 - v_x}{1}$$

$$\therefore I_1 = v_1 \left[\frac{1}{2} + 1 \right] - v_x = \frac{3}{2} v_1 - v_x$$

$$\boxed{I_1 = \frac{3}{2} v_1 - \frac{1}{2} v_x} \quad (2)$$

$$I_5 = \frac{v_x - v_1}{4} = \frac{v_1 - v_2}{2}$$

$$\frac{v_x}{4} - \frac{v_1}{4} = \frac{v_1}{2} - \frac{v_2}{2}$$

$$\therefore \boxed{v_x = 3v_1 - 2v_2} \quad (3)$$

from (3) in (2) :-

$$I_1 = 2v_2 - \frac{3}{2} v_1$$

$$\text{We put } \boxed{I_1 = \frac{v_{in} - v_1}{2}}$$

$$\therefore \frac{v_{in} - v_1}{2} = 2v_2 - \frac{3}{2} v_1$$

$$\therefore \boxed{v_{in} = 4v_2 - 2v_1} \quad (4)$$

apply KCL at node ③ :-

$$I_2 + I_5 = I_6$$

$$\frac{v_{in} - v_2}{1/s_c} + \frac{v_1 - v_2}{2} = \frac{v_2}{2}$$

$$v_{in} s_c - v_2 s_c + \frac{1}{2} v_1 - \frac{1}{2} v_2 = \frac{1}{2} v_2$$

$$\therefore v_{in} s_c = v_2 \left[1 + s_c \right] + \frac{1}{2} v_1$$

$$\boxed{v_{in} = v_2 \left[\frac{1}{s_c} + 1 \right] - \frac{1}{2s_c} v_1} \quad (5)$$

we equating equation (4) and (5) :-

$$4v_2 - 2v_1 = \frac{1}{s_c} v_2 + v_2 - \frac{1}{2s_c} v_1$$

$$2v_1 - \frac{1}{2s_c} v_1 = 3v_2 - \frac{1}{s_c} v_2$$

$$4s_c v_1 - v_1 = 6s_c v_2 - 2v_2$$

$$v_1 [4s_c - 1] = v_2 [6s_c - 2]$$

$$V_2 = V_1 \frac{4sC - 1}{6sC - 2} \quad (6)$$

From (6) in (4) :-

$$V_{in} = V_1 \left[\frac{16sC - 4}{6sC - 2} - 2 \right]$$

$$\therefore V_{in} = V_1 \left[\frac{16sC - 4 - 12sC + 4}{6sC - 2} \right]$$

$$V_{in} = V_1 \left[\frac{4sC}{6sC - 2} \right]$$

$$\therefore V_1 = V_{in} \left[\frac{6sC - 2}{4sC} \right] = V_{in} \left[\frac{3}{2} - \frac{2}{4sC} \right] \quad (7)$$

From (7) in (1) :-

$$I_{in} = V_{in} \left[\frac{1}{2} + sC \right] - \frac{1}{2} V_1 \left[\frac{3}{2} - \frac{2}{4sC} \right] - sC V_{in} \left[\frac{3}{2} - \frac{2}{4sC} \right] \left[\frac{4sC - 1}{6sC - 2} \right]$$

$$= V_{in} \left[\frac{1}{2} + sC \right] - V_{in} \left[\frac{3}{4} - \frac{1}{4sC} \right] - V_{in} \left[\frac{3}{2} sC - \frac{1}{2} \right] \left[\frac{4sC - 1}{6sC - 2} \right]$$

$$\therefore \frac{I_{in}}{V_{in}} = -\frac{1}{4} + sC + \frac{1}{4sC} - \left[\frac{12s^2C^2 - 7sC + 1}{12sC - 4} \right]$$

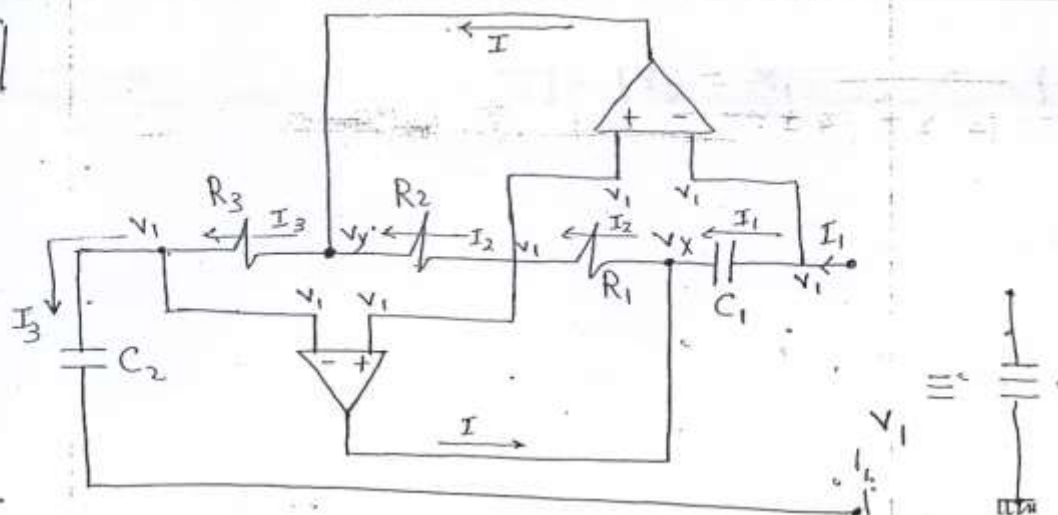
$$= -\frac{1}{4} + sC + \frac{1}{4sC} - \left[\frac{(4sC - 1)(3sC - 1)}{4(3sC - 1)} \right]$$

$$= -\cancel{\frac{1}{4}} + \cancel{sC} + \frac{1}{4sC} - \cancel{sC} + \cancel{\frac{1}{4}} = \frac{1}{4sC}$$

$$\therefore \frac{V_{in}}{I_{in}} = Z_{in} = 4sC = sL$$

$$\therefore \text{where } [L = 4C] \quad \#$$

4



$$I_1 = \frac{V_1 - V_x}{1/sC_1} = (V_1 - V_x) sC_1 \rightarrow (1)$$

$$I_2 = \frac{V_x - V_1}{R_1} = \frac{V_1 - V_y}{R_2}$$

$$\therefore \frac{V_1 - V_x}{R_1} = \frac{V_y - V_1}{R_2} \rightarrow (2)$$

From (2) in (1) :-

$$\frac{I_1}{sC_1 R_1} = \frac{V_y - V_1}{R_2}$$

$$\therefore V_y - V_1 = \frac{R_2 I_1}{sC_1 R_1} \rightarrow (3)$$

$$I_3 = \frac{V_y - V_1}{R_3} = \frac{V_1}{1/sC_2} = V_1 sC_2 \rightarrow (4)$$

$\therefore$ divide equation (3) by (4) :-

$$\frac{V_y - V_1}{R_3} = \frac{R_2 I_1}{sC_1 R_1 R_3} \rightarrow (5)$$

From (4) in (5) :-

$$V_1 sC_2 = \frac{R_2 I_1}{sC_1 R_1 R_3} \rightarrow \left[\frac{V_1}{I_1} = Z_{in} = \frac{R_2}{s^2 C_2 C_1 R_1 R_3} \right] \rightarrow \underline{\underline{FDNR}}$$